Philosophy of Mathematics

Department: **Fudan International Summer Session 2025**

<table>
<tr><td>Course Code</td><td colspan="7">PHIL130113</td></tr>
<tr><td>Course Title</td><td colspan="7">Philosophy of Mathematics</td></tr>
<tr><td>Credit</td><td>2</td><td>Experiment (including Computer) Credit</td><td>0</td><td>Practice Credit</td><td>0</td><td>Aesthetic Education Credit</td><td>0</td></tr>
<tr><td>Credit Hours Per Week</td><td>9 Credit hours per week, 36+3 credit hours in total</td><td>Education on The Hard-Working Spirit Credit Hours</td><td></td><td>Language of Instruction</td><td>English</td><td>Honors Course</td><td>□Yes
■No</td></tr>
<tr><td rowspan="2">Course Type</td><td colspan="3" rowspan="2">□Core General Education Course
□Specific General Education Course
□Basic Course in General Discipline
■Others</td><td colspan="4">2+X Major ：
□Professional Core Course
□Professional Advanced Course</td></tr>
<tr><td colspan="4">Non 2+X Major ：
□Professional Compulsory Course
■Professional Elective Course</td></tr>
<tr><td>Course Objectives</td><td colspan="7">Value: To cultivate a critical and philosophical appreciation for the foundational nature of mathematics, its unique methods, and its relationship to other fields of knowledge. To explore fundamental questions about the nature of mathematical objects, truth, and knowledge.
Knowledge: To gain familiarity with central topics and debates in the philosophy of mathematics, including Platonism, logicism, structuralism, and the nature of infinity, proof, and computability. To understand the role of set theory, the phenomena of independence in contemporary mathematics.
Ability: To develop skills in philosophical analysis and argumentation, applied to mathematical concepts and practices. To critically evaluate different philosophical perspectives on mathematics and to articulate one's own informed views. To enhance capacities in discussing abstract concepts.</td></tr>
<tr><td>Course Description</td><td colspan="7">This course provides a comprehensive introduction to the philosophy of mathematics, exploring fundamental questions about the nature of mathematical objects, mathematical truth, and mathematical knowledge. We will examine a range of historical and contemporary perspectives, including Platonism, logicism, and structuralism, and delve into key topics such as the nature of numbers, infinity, rigor, proof, computability, and incompleteness. The course will also address modern challenges and developments, including the foundational roles of set theory and type theory, as well as the implications of proof assistants and artificial intelligence for mathematics. Special attention will be</td></tr>
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	given to set theory, with discussion in contemporary set theoretic topics such as large cardinals, forcing, and the multiverse view. The role of axioms will be discussed.

Course Requirements:

Active Participation: Students are expected to attend all lectures and actively engage in class discussions. This includes asking questions, contributing to debates, and respectfully responding to the views of others.

Reading Assignments: Students are required to complete all assigned readings before each lecture. The readings will form the basis for class discussions and the final presentation. Close and critical reading is essential.

Presentation Preparation: Students will prepare and deliver one presentation (individual or group, as determined by the instructors) on a chosen topic related to the course material. The presentation should demonstrate a thorough understanding of the selected readings and offer a clear, well-argued philosophical perspective.

No previous knowledge of philosophy or advanced mathematics is assumed or required.

Teaching Methods:

Lectures: The primary mode of instruction will be lectures, delivered by the course instructors, introducing key concepts, historical context, and philosophical arguments.

Guided Discussion: Lectures will be interspersed with periods of guided discussion, where students will be encouraged to critically analyze the material, raise questions, and debate different viewpoints.

Presentation and Feedback: Student presentations will provide an opportunity for in-depth exploration of specific topics and will be followed by constructive feedback from instructors and peers.

Course Director's Academic Background:

Ruizhi Yang is an Associate Professor in the School of Philosophy at Fudan University. His research centers on the philosophy of mathematics, set theory, and computability theory. He received his Ph.D. in Logic from the Department of Philosophy at Peking University in 2012. Dr. Yang has held visiting positions at Harvard University (2010-2011) and the City University of New York (CUNY) (2017-2018).

Instructor's Academic Background:

Joel David Hamkins is the John Cardinal O'Hara Professor of Logic at the University of Notre Dame. A leading figure in mathematical and philosophical logic, his research focuses on set theory, the mathematics and philosophy of the infinite, large cardinals, and infinitary computability. He earned his Ph.D. in Mathematics from the University of California, Berkeley in 1994. Prior to joining Notre Dame, Professor Hamkins held the position of Professor of Logic at Oxford University and was the Sir Peter Strawson Fellow at University College, Oxford (2018-2022). He also held a long-standing professorship at The City University of New York (CUNY).

Members of Teaching Team

Name	Gender	Professional Title	Department	Responsibility

Ruizhi Yang	Male	Associate Professor	Philosophy	Director, Instructor, Lecturer
Joel David Hamkins	Male	Professor	Philosophy	Instructor, Lecturer
Zhaokuan Hao	Male	Professor	Philosophy	Instructor, Lecturer

Course Schedule (Please supply the details about each lesson)**:**

Lecture 1: General introduction to the philosophy of mathematics.

How does mathematics pose a philosophical challenge?

Lecture 2: Modern challenge to the philosophy of mathematics.

The fundamental roles of set theory, category theory, and type theory; proof assistant programming languages; and artificial intelligence in mathematics.

Lecture 3: Numbers.

What are numbers in different number systems (e.g., natural numbers, real numbers, transcendental numbers)? Platonist, logicalist, and structuralist views on numbers.

Lecture 4: Rigor.

Continuity; instantaneous change (infinitesimals and the modern definition of the derivative); the indispensability of mathematics; abstraction.

Lecture 5: Infinity.

Equinumerous; countable infinity; uncountability; the cardinality of the power set of a set; Cantor's continuum hypothesis; Zeno's paradoxes.

Lecture 6: Geometry.

Constructions; non-constructability; the ontology of geometry; non-Euclidean geometry; geometry and physical space.

Lecture 7: Proof.

Syntax and semantics; proofs in working mathematics; formal proofs and proof theory; automated theorem proving; completeness; non-classical logics.

Lecutre 8. Computability.

Primitive recursion; Turing machines; computational power; computation in the physical universe; undecidability; oracle computing; complexity theory.

Lecture 9. Incompleteness.

The Hilbert Program; the First Incompleteness Theorem; the Second Incompleteness Theorem; Tarski's Theorem on the Undefinability of Truth; the ubiquity of independence; reverse mathematics.

Lecture 10. Set theory.

The Cantor-Bendixson Theorem; set theory as a foundation of mathematics; the comprehension principle; the cumulative hierarchy; the separation axiom and ill-founded hierarchies; extensionality; the replacement axiom; the Axiom of Choice; large cardinals; forcing; the multiverse view.

Discussion and examination.

One or two sections of student presentations (to be determined based on enrollment). If only one section is sufficient, we will present an additional lecture on set theory and the infinite.

The design of class discussion or exercise, practice, experience and so on:

In-class discussions and one group or individual presentation.

If you need a TA, please indicate the assignment of assistant:

Collecting feedback and answering questions; organizing discussions and taking notes; assisting with evaluation.

Grading & Evaluation (Provide a final grade that reflects the formative evaluation process)**:**

In-class discussion 40%: Students are encouraged to participate by asking questions and presenting their views.

Final presentation 60%: The presentation should reflect the readings and the student's reflections.

Usage of Textbook： ■Yes(complete textbook information form below) □No

Textbook Information (No more than two textbooks) **:**

Title	Author	ISBN	Publishing Time	Publisher	Type I	Type II
Lectures on the Philosophy of Mathematics	Joel David Hamkins	978-0-262-54223-4	2020	The MIT press	■Self-compiled Textbook (Published) ■Non-mainland Textbook □Other Textbook (Published)	□National Planning Textbook □Provincial and Ministerial Planning Textbook □School Level Planning Textbook □Others

Teaching References (Including author, title, publisher, publishing time,ISBN)**:**

Øystein Linnebo, *Philosophy of Mathematics*, 2017, Princeton University Press.ISBN: 978-0-691-16140-2.

Table column size can be adjusted according to the content.